UNIVERSITY OF CAMBRIDGE **MPhil in Population Health Sciences**

Self-Screening Questions for Health Data Science

Version 0.9

A note from the admissions committee

This worksheet is for those considering an application to study the Health Data Science stream of the MPhil in Population Health Sciences at the University of Cambridge. Mathematical skills typical of an excellent A-level student are a minimum requirement for applicants to this stream. The questions are designed to allow the potential applicant to assess for her/himself whether s/he is likely to be able to meet the mathematical standard when s/he begins the course, if s/he is offered a place.

As you work through the questions, it is important that you are not put off if you find some of them challenging. We appreciate that many applicants, particularly those with a medical or biological background, may not have studied mathematics for some time. The aim is not to test whether you are ready to tackle the MPhil course today, but to give you the opportunity to assess whether you will be able to put yourself in the position to start the course with the level of mathematics required. If you find a question difficult, please look up the definitions of any terms of which you are unsure, study the resources suggested and return to the question. If you find you are unable to make progress after much further study you may wish to refer to the solutions, but we suggest that you consider doing so as a last resort. If you refer to the solutions, bear in mind that there may be other approaches to an answer that are correct. We hope you will view this as an opportunity to learn or review some mathematics, and, if you apply successfully, to ensure you are ready to take the course when you arrive in Cambridge.

If you have a strong interest in studying population health sciences but find many of these questions difficult (i.e. if you need to refer to many of the solutions even after further study) you may wish to consider applying to a stream with a less stringent mathematical requirement. For example, you could consider the Epidemiology stream, which is less mathematically demanding than the Health Data Science stream, but nevertheless provides a quantitative training and includes courses in biostatistics. A mathematical self-screening tool for the other streams is available from the the course website.

If you find an error please report to wja24@cam.ac.uk.

Question. Graphs

Each equation corresponds to a graph. Identify each correspondence and give your reasoning.

(a) $y = x + 1$

(b) $y = -x + 1$

(c) $y = \frac{2}{3}x^2 - \frac{8}{3}x + 2$

(d) $y = -\frac{1}{30}(x-1)(x+61)$

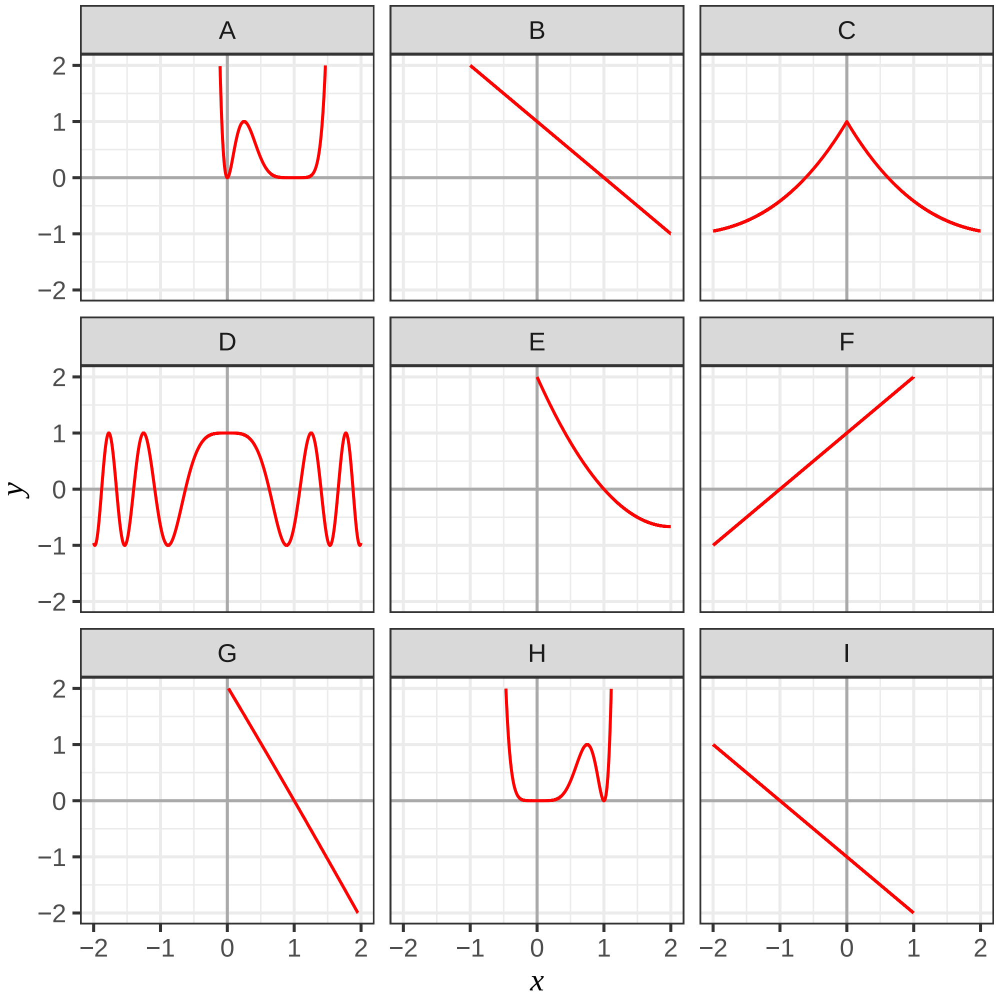
(e) $y = \dfrac{2^{16}}{3^6}x^6(1-x)^2$

(f) $y = \dfrac{2^{16}}{3^6}x^2(1-x)^6$

(g) $y = \dfrac{x^2}{\log(e^{-x})} - 1$

(h) $y = \sin(4x^2 + \pi/2)$

(i) $y = \cos(2\sqrt{|x|})$



Question. Regions

Of the square in the (x, y) plane satisfying,

$$-5 < x < 5$$
$$-5 < y < 5,$$

sketch the sub-region determined by each of the inequalities.

(a) $x < 2y + 1$

(b) $xy < 1$

(c) $|x||y - 1| < 4$

(d) $1 < y$ and $(x - 2)x + y < 4$

(e) $(x + 1)^2 + (y - 2)^2 < 2|x + y - 1| + 2$

Question. Exponentials and Logarithms

The exponential function, $x \mapsto e^x$, is monotonic and strictly increasing. For every pair of numbers (a,b), it satisfies the identities:

$$e^a e^b = e^{a+b}$$
$$(e^a)^b = e^{ab}.$$

The natural logarithm function $x \mapsto \log x$ is the inverse of the exponential function, i.e. $\log x$ is such that $e^{\log x} = x$, for every $x > 0$.

Of the following statements, determine which are true and which are false for all $x > 0$ and all $y > 0$.

(a) $\dfrac{1}{(e^x)^y} = e^{-xy}$

(b) $2e^{x-y} - 2e^{y-x} = (e^x - e^{-x})(e^y + e^{-y}) - (e^x + e^{-x})(e^y - e^{-y})$

(c) $\dfrac{e^{y+x-2}}{e^{x^2+x-6}} = e^{y-x+3}$

(d) $y\dfrac{e^x}{\sqrt{x}} = e^{\log y + x - \frac{1}{2}}$

(e) $y^{x+\frac{1}{8}} = e^{x \log y}\sqrt{\sqrt{\sqrt{y}}}$

(f) $\dfrac{x}{y} = e^{\log_y x}$

(g) $\log(x + y) = \log x \log y$

(h) $\log x + \log y = \log(xy)$

(i) $x^{\log y} = y^{\log x}$

(j) $\log_x(y) = \dfrac{\log x}{\log y}$

(k) $\log_{1/x}(y) = -\dfrac{\log y}{\log x}$